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Analytical description of heavy ion potentials for collisions between nuclei of same shell

Manoj K. Sharma[1,2], Rajeev K. Puri[2], Raj K. Gupta[2]

[1] Adesh Institute of Engineering and Technology, Faridkot, India.
[2] Center of Advanced Study in Physics, Panjab University, Chandigarh-160014, India.

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Abstract. Using Skyrme energy density formalism, we present an analytical formula of ion-ion potential (including spin-density part) in terms of the masses of colliding nuclei. The parametrization of the spin-independent part of the ion-ion potential is based on the proximity theorem whereas the spin-dependent potential is parametrized in terms of "the masses of colliding nuclei and their associated particle strength". The particle strength accounts for the number of valence particles outside the closed core. Adding Coulomb interaction, this parametrization of ion-ion potential introduces a great simplification for the calculation of fusion barriers and cross-sections analytically. Our parametrized potentials are in good agreement with other theoretical potentials and the fusion cross-sections calculated with this potential are in good agreement with experimental data.

PACS.

I Introduction

In last several years, lot of work has been done in low energy heavy ion physics. In this energy domain, the **S**kyrme **E**nergy **D**ensity **F**ormalism (SEDF) has proven to be quite successful in explaining the phenomena like interaction barriers, fusion cross-sections, elastic scattering, nucleon transfer processes, etc. [1-9]. The same Skyrme energy density formalism is also used extensively for heavy ion collisions at intermediate and relativistic energies [10−12].

One of the important questions at low energy is whether we can understand the nucleus -nucleus potential in terms of some fundamental quantities like masses and charges of colliding nuclei or not. These quantities are always known in any experiment and therefore, such parametrization of the nucleus-nucleus potential can be of great importance for comparing the theoretical predictions directly with experimental results. The present study deals with a parametrization of the ion-ion interaction potential in terms of some fundamental quantities like masses and charges of colliding nuclei at low energy. In the past, several successful attempts have been made to parametrize the *spin density-independent* ion-ion potentials [2,7,8]. By neglecting the spin-density part of the heavy ion potential, one is reducing the scope of formalism to spin saturated nuclei (like ^{16}O, ^{40}Ca etc.) only. On the other hand, we know that the contribution of spin-density part towards fusion cross-section can be as much as 50 mb [5]. Therefore, an apparent need is to obtain a general analytical formula of the potential for both the spin-saturated and spin-unsaturated colliding nuclei.

The analyticity of the spin-saturated potential is guided by the proximity concept and hence the parametrization is straight forward. The proximity theorem states that the interaction potential $V = 2\pi\bar{R}\Phi(s)$, where $\Phi(s)$ is an universal function and $\bar{R}$ is responsible for the geometry of colliding nuclei. One can parametrize the $\Phi(s)$ uniquely and hence the spin-saturated nuclear potential. On the contrary, the parametrization of the spin-density dependent part of the ion-ion potential is quite tedious. This is because it involves the angular momenta and the intrinsic spins of nucleons. Furthermore, the magnitude of shell model radial wave function depends strongly on the orbital angular momentum. Therefore, one is left with no other option but to parametrize the spin-density potential of colliding nuclei belonging to the same shell, for both the protons and neutrons.

In this paper, we present a complete parametrization of the ion-ion interaction potential (including spin-density part) in terms of masses and charges of colliding nuclei. The present work is an extension of our earlier work [4], to include collisions between all shells up to 0g and consider another set of parameters for the Skyrme force. We also compare our parametrized potentials with other theoretical potentials. Some results of fusion cross-sections are also given.

The article is organised as follows: Section II deals with a brief outline of the energy density formalism. The

analytical parametrization along with some results is presented in Sect. *III* and our results summarized in Sect. *IV*.

II The model

The starting point of our model is the S kyrme E nergy D ensity F unctional $H(\boldsymbol{r})$. The volume integration of energy density functional $H(\boldsymbol{r})$ leads to energy expectation value [3] :

$$E = \int H(\rho, \tau, \boldsymbol{J}) d\boldsymbol{r}. \tag{1}$$

The interaction potential then reads as:

$$\begin{aligned} V_N(R) &= E(R) - E(\infty) \\ &= \int [H(\rho, \tau, \boldsymbol{J}) - H_1(\rho_1, \tau_1, \boldsymbol{J}_1) \\ &\quad - H(\rho_2, \tau_2, \boldsymbol{J}_2)]\, d\boldsymbol{r}. \end{aligned} \tag{2}$$

with $\rho(=\rho_n+\rho_p)$, $\tau(=\tau_n+\tau_p)$ and $\boldsymbol{J}(=\boldsymbol{J_n}+\boldsymbol{J_p})$, being the nucleon density, the kinetic energy density and the spin density, respectively. Here indices n and p are the neutrons and protons, respectively. Under the sudden approximation, we have $\rho = \rho_1+\rho_2,\ \tau = \tau_1+\tau_2,\ \boldsymbol{J} = \boldsymbol{J_1}+\boldsymbol{J_2}$. In other words, the interaction potential $V_N(R)$ is the difference of energy expectation value E of two colliding nuclei at a finite distance R and at a completely separated distance ($R = \infty$).
Following Vautherin and Brink [13], the energy density functional $H(\boldsymbol{r})$ is given by :

$$\begin{aligned} &H(\rho, \tau, \boldsymbol{J}) = H(\rho, \tau) + H(\rho, \boldsymbol{J}) \\ &= \left\{ \frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0[(1+\frac{1}{2}x_0)\rho^2 - (x_0+\frac{1}{2})(\rho_n^2+\rho_p^2)] \right. \\ &\quad + \frac{1}{4}(t_1+t_2)\rho\tau + \frac{1}{8}(t_2-t_1)(\rho_n\tau_n+\rho_p\tau_p) \\ &\quad + \frac{1}{16}(t_2-3t_1)\rho\nabla^2\rho \\ &\quad \left. + \frac{1}{32}(3t_1+t_2)(\rho_n\nabla^2\rho_n+\rho_p\nabla^2\rho_p) + \frac{1}{4}t_3\rho_n\rho_p\rho \right\} \\ &\quad + \left\{ -\frac{1}{2}W_0(\rho\boldsymbol{\nabla}\cdot\boldsymbol{J} + \rho_n\boldsymbol{\nabla}\cdot\boldsymbol{J}_n + \rho_p\boldsymbol{\nabla}\cdot\boldsymbol{J}_p) \right\}. \end{aligned} \tag{3}$$

In (3), the six parameters t_0, x_0, t_1, t_2, t_3 and W_0 are fitted by different authors to obtain a better description of the various ground-state properties of nuclei. Here we use the Skyrme force SIII with $t_0 = -1128.75 MeV fm^3$, $t_1 = 395 MeV fm^5$, $t_2 = -95 MeV fm^5$, $t_3 = 14000 MeV fm^6$, $W_0 = 120 MeV fm^5$ and $x_0 = 0.45$.

The kinetic energy density appearing in (3) can be written as a function of nucleonic density, including the von Weizsacker surface correction as : $\tau = \frac{3}{2}(\frac{3}{2}\pi^2)^{\frac{2}{3}}\rho^{\frac{5}{3}} + \lambda\frac{(\boldsymbol{\nabla}\rho)^2}{\rho}$. The value of constant λ has been a point of controversy and hence in present calculations, we put $\lambda = 0$.

Now the energy density functional $H(\boldsymbol{r})$ in (3) depends on functions of ρ and $\boldsymbol{J}$ only and it is possible to separate the interaction potential in to spin-independent and spin-dependent parts:

$$\begin{aligned} V_N(R) &= \int \{H(\rho) - [H_1(\rho_1) + H_2(\rho_2)]\}\, d\boldsymbol{r} \\ &\quad + \int \{H(\rho, \boldsymbol{J}) - [H_1(\rho_1, \boldsymbol{J_1}) + H_2(\rho_2, \boldsymbol{J_2})]\}\, d\boldsymbol{r} \\ &= V_P(R) + V_J(R). \end{aligned} \tag{4}$$

Following [3,4,14], $V_P(R)$ can be calculated in the spirit of proximity force theorem of [15],

$$V_P(R) = 2\pi\bar{R}\Phi(s) = 2\pi\bar{R}\int e(s) ds \tag{5}$$

where

$$\Phi(s) = \int \{H(\rho) - [H_1(\rho_1) + H_2(\rho_2)]\}\, dZ \tag{6}$$

and

$$\begin{aligned} &\bar{R} = \frac{C_1 C_2}{C_1 + C_2} \quad \text{with} \quad C_i = R_i - \frac{1}{R_i} \\ &\text{and} \quad R_i = 1.28 A_i^{\frac{1}{3}} - 0.76 + 0.8 A_i^{-\frac{1}{3}}. \end{aligned}$$

The separation distance $s = R - C_1 - C_2$ and $e(s)$ is the interaction energy per unit area between two flat slabs of semi-infinite nuclear matter with surfaces parallel to X-Y plane and moving in Z-direction. Clearly $\int e(s) ds$ in (5) does not depend on the geometry of colliding nuclei and hence is an universal function. For full details, we refer the reader to original [3,14].

For nucleonic density, we use two parameter Fermi density:

$$\rho_i(r_i) = \rho_{0i}\left[1 + \exp\frac{(r_i - R_{0i})}{a_i}\right]^{-1}, \quad i = 1, 2. \tag{7}$$

The values of constants a_i, R_{0i} and ρ_{0i} can be found in ref. [3]. The spin-dependent part of the interaction potential reads as

$$V_J(R) = -\frac{3}{4}W_0 \int [\rho_2(\boldsymbol{\nabla}\cdot\boldsymbol{J}_1) + \rho_1(\boldsymbol{\nabla}\cdot\boldsymbol{J}_2)]\, d\boldsymbol{r}, \tag{8}$$

where Vautherin and Brink [13] defines the spin-density $\boldsymbol{J}$ as

$$\boldsymbol{J}_q(\boldsymbol{r}) = (-i)\sum_{i,s,s'} \phi_i^*(\boldsymbol{r}, s, q)\, [\boldsymbol{\nabla}\phi_i(\boldsymbol{r}, s', q) \times < s|\boldsymbol{\sigma}|s' >]\,. \tag{9}$$

Here s and q represent the spin and isospin indices, respectively and the summation i runs over all the occupied single particle orbitals. For ϕ_i, an ansatz has been used

$$\phi_i(\boldsymbol{r},s,q) = \frac{R_\alpha(r)}{r}\sum_{m_l m_s} < l\frac{1}{2}m_l m_s|jm > Y_l^{m_l}(\hat{r})\chi_{m_s}(s)\chi_q(t), \quad (10)$$

with $\alpha = q,n,l$ and the normalized shell model radial wave functions

$$R_{nl}(r) = C_{nl}r^{l+1}e^{-\nu r^2}\nu_{nl}(2\nu r^2), \quad (11)$$

with

$$C_{nl} = \left[\frac{[2^{l-n+2}(2\nu)^{l+\frac{3}{2}}(2l+2n+1)!!]}{\sqrt{\pi}[(2l+1)!!]^2 n!}\right]^{1/2}, \quad (12)$$

$$\nu_{nl}(x) = \sum_{k=0}^{n}(-1)^k 2^k \begin{bmatrix} n \\ k \end{bmatrix}\frac{(2l+1)!!}{(2l+2k+1)!!}x^k, \quad (13)$$

and

$$2\nu = \frac{41A^{-(1/3)}mc^2}{\hbar^2 c^2} \qquad (\text{in fm}^{-2}). \quad (14)$$

For an even-even nucleus with valence particles (or holes) outside (inside) the closed core, Puri et al. [3] divided the $\boldsymbol{J}_q(r)$ into two parts (for $q = n$ or p)

$$\boldsymbol{J}_q(r) = \boldsymbol{J}_q^c(\boldsymbol{r}) \pm \boldsymbol{J}_q^{n_v}(\boldsymbol{r}). \quad (15)$$

Here, the first term gives the contribution due to core consisting of closed shells and the second term due to valence n_v particles (or holes) with (+) for particles and (−) for holes. $\boldsymbol{J}_q^c(\boldsymbol{r})$ and $\boldsymbol{J}_q^{n_v}(\boldsymbol{r})$ reads as:

$$\boldsymbol{J}_q^c(\boldsymbol{r}) = \frac{\boldsymbol{r}}{4\pi r^4}\sum_\alpha (2j_\alpha+1) \left[j_\alpha(j_\alpha+1) - l_\alpha(l_\alpha+1) - \frac{3}{4}\right]R_\alpha^2(r) \quad (16)$$

and

$$\boldsymbol{J}_q^{n_v}(\boldsymbol{r}) = \frac{n_v\boldsymbol{r}}{4\pi r^4}[j(j+1) - l(l+1) - \frac{3}{4}]R_l^2(r). \quad (17)$$

Naturally, if core is a major-shell (i.e. orbits with $j = l \pm \frac{1}{2}$ pairs fully occupied), the contribution of the spin density part of the ion-ion potential will be zero. The nucleus-nucleus potential calculated by using the above formalism will be refered to as the "exact" potential, in the following.

III Analytical formulation of the heavy ion interaction potential

Here we consider a large number of collisions involving nuclei from different shells. We restrict ourselves to nuclei belonging to the same shell. The colliding nuclei can be as light as $^{12}C+^{12}C$ (0s-0p shells) and as heavy as $^{116}Ce+^{116}Ce$ ($0g-0g$ shells). We first point out the limitations of our model, which influence our parametrization.

(i). Both even and odd nuclei are involved in low energy phenomena like fusion of colliding nuclei, particle transfer reactions, cluster radioactive decay of nuclei, fission, etc. However, SEDF is limited to even-even nuclei only.

(ii). The exotic cluster decay study involves the emission of light nuclei such as α, ^{14}C, ^{20}O, etc. We know that in light nuclei, the surfaces are comparable to their cores. Therefore, the proximity concept (used in the present study) becomes a poor approximation and hence we exclude the 0s-0p shells from our parameterization.

(iii). We know that for colliding nuclei, the depth of potential pocket becomes shallower for heavier nuclei which eventually disappears for very heavy nuclei. Therefore, parametrization of the ion-ion interaction potential for nuclei belonging to shells like $0h_{\frac{9}{2}}$ and higher, does not make much sense. Hence in the present study, we parametrize the ion-ion interaction potential for colliding nuclei belonging to (1s-0d), 0f, 1p, and 0g shells only. In other words, our present parameterization is valid for the colliding nuclei with $8 \leq\ Z \leq 58$.

In the following, we first discuss the analytical formula for V_P (the spin- independent part of the interaction potential) and then give the details of how one can parameterize the spin- dependent part V_J of the interaction potential.

A Analytical formulation of the universal function $\Phi(s)$.

From (5), it is clear that one can separate the proximity potential $V_P(R)$ into a product of universal function $\Phi(s)$ (which is independent of the geometry of colliding nuclei) and a geometrical factor (which depends on the radii of colliding nuclei). We have calculated the universal

Table 1. Calculated values of Φ_0 and s_0 for some target-projectile combinations using SIII $\lambda = 0$

System	Φ_0 ($\frac{MeV}{fm}$)	s_0 (fm)	System	Φ_0 ($\frac{MeV}{fm}$)	s_0 (fm)
$^{18}O+^{36}Ar$	2.267	0.3	$^{18}O+^{24}Mg$	2.262	0.3
$^{20}Ne+^{30}Si$	2.321	0.3	$^{20}Ne+^{34}S$	2.315	0.3
$^{22}Ne+^{26}Mg$	2.323	0.3	$^{34}S+^{32}S$	2.318	0.3
$^{34}S+^{36}Ar$	2.317	0.3	$^{30}Si+^{34}S$	2.325	0.4
$^{42}Ca+^{64}Ge$	2.312	0.2	$^{42}Ca+^{58}Ni$	2.307	0.2
$^{48}Cr+^{62}Zn$	2.310	0.2	$^{48}Cr+^{64}Ge$	2.310	0.2
$^{46}Ti+^{60}Zn$	2.309	0.2	$^{46}Ti+^{68}Se$	2.310	0.2
$^{54}Fe+^{60}Zn$	2.311	0.1	$^{72}Kr+^{74}Kr$	2.304	0.2
$^{70}Se+^{70}Se$	2.308	0.2	$^{70}Se+^{76}Sr$	2.299	0.2
$^{72}Kr+^{80}Zr$	2.307	0.3	$^{74}Kr+^{76}Sr$	2.307	0.3
$^{76}Sr+^{76}Sr$	2.308	0.3	$^{78}Sr+^{80}Zr$	2.308	0.3
$^{98}Sn+^{102}Te$	2.301	0.4	$^{96}Sn+^{92}Sn$	2.304	0.4
$^{88}Mo+^{110}Ba$	2.303	0.4	$^{86}Mo+^{84}Mo$	2.306	0.3
$^{92}Pd+^{88}Pd$	2.303	0.3	$^{90}Mo+^{90}Mo$	2.303	0.3
$^{106}Xe+^{110}Ba$	2.303	0.4	$^{118}Ce+^{122}Nd$	2.301	0.4

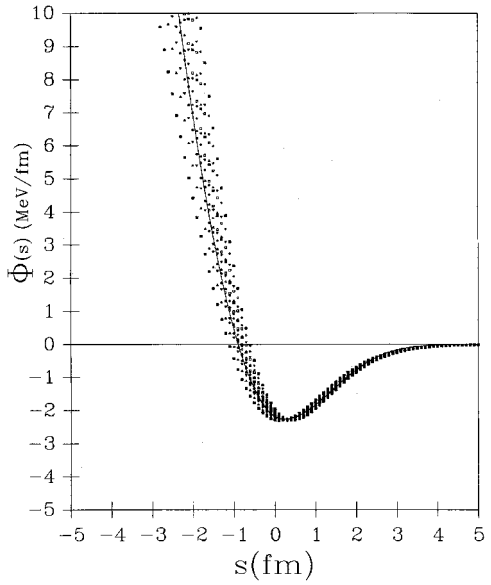

Fig. 1. Universal function $\Phi(s)$ as a function of distance s. Different symbols represent exact universal function whereas solid line is our parametrized form

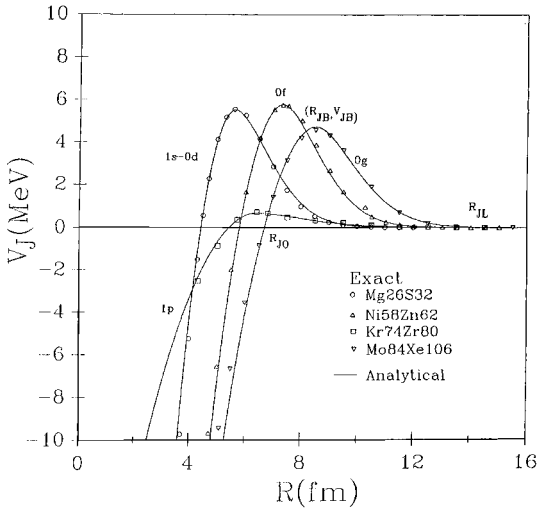

Fig. 2. The spin density potential V_J as a function of relative separation distance R

function (eq. 6) for nearly 200 reactions. In Table 1, the minimum values of $\Phi(s)$ (i.e. Φ_0) along with its position s_0 are given for some 30 typical reactions. From Table 1, one can conclude that for medium and heavy colliding nuclei, the Φ_0 and s_0 are nearly constant. This finding is the same as has been reported in ref. [4] for another Skyrme force and has been discussed by different authors [2, 7, 8].

The universal function $\Phi(s)$ for some of the colliding nuclei is plotted in Fig 1 as a function of s. We find that the scattering of different $\Phi(s)$ from a mean behaviour is quite small. For ($s \leq s_0$), this scattering is larger, but this part of the interaction potential has no physical importance. Firstly, the sudden approximation is not valid for the overlaping regions and secondly, the measurable quantities, like fusion barriers/cross-sections, occur at distance $R > R_{touching} (= R_1 + R_2, R_i$ being the radii of colliding nuclei). In this region of the interaction potential, the scattering in $\Phi(s)$ is quite small. The general behaviour of $\Phi(s)$ in Fig.1 can be parametrized as :

$$\Phi(s) = \left\{ \begin{array}{ll} -\Phi_0 \ \exp\left[-0.3325(s-s_0)^2\right], & \text{for s} \ \geq \ \text{s}_0, \\ -\Phi_0 + 1.90(s-s_0)^2, & for \ s \ \leq s_0 \end{array} \right\}, \quad (18)$$

with $\Phi_0 = 2.27$ (MeV/fm) and $s_0 = 0.2$ fm. In Fig. 1, this parametrization is shown by a solid line. Similiar plots and parameterizations have also been carried out in Refs. [2, 4, 7, 8]. The spin - independent ion-ion potential V_P can be obtained by multiplying $\Phi(s)$ with geometrical factor $2\pi\bar{R}$ of the colliding nuclei. Note that in parametrizing the universal function $\Phi(s)$, no restriction of the shells is imposed on colliding pairs. In other words, this parametrization of V_P(R) is independent of the shell picture of the colliding nuclei. The only restrictions imposed are (i) the nuclei should not be too light (because the proximity theorem is better suited for medium and heavy nuclei only) and (ii) the colliding partners should have even-even masses.

B Analytical formulation of the spin-dependent part V_J of the potential.

In contrast to the proximity part of the interaction potential, we cannot split the spin-dependent part of potential into a geometrical factor and an universal function. This is because the spin density $\boldsymbol{J}(\boldsymbol{r})$, appearing in V_J (eq. 8), depends on the radial wave function $R_{nl}(r)$ which varies from shell to shell [4]. The radial wave function is nearly constant or varies smoothly within a shell. Therefore, we have to restrict our parametrization of the spin density potential to the colliding nuclei belonging to the same shell. First such attempt was made by Puri and Gupta [4]. They restricted themselves to (1s-0d) and ($0f_{7/2} - 0f_{7/2}$) shells only. Here, we parametrize the V_J for several shells (from 1s-0d to 0g) in terms of valence particles and masses of colliding nuclei.

In Fig. 2, we show the spin density dependent part of the ion-ion potential (symbols) as a function of separation distance R for the collisions of nuclei belonging to 1s-0d, 0f, 1p and 0g shells. We find that the spin density part of the potential behaves opposite to the proximity potential (compare Figs. 1 and 2). As we are trying to parameterize the potential within a shell, we assume that 0f and 1p shells donot intermix with each other i.e. ($0f_{\frac{7}{2}} - 0f_{\frac{5}{2}}$) shells are filled first and then ($1p_{\frac{3}{2}} - 1p_{\frac{1}{2}}$) shells are filled. In other words, we assume that the $0f_{\frac{5}{2}}$ shell lies lower in the energy than the $1p_{\frac{3}{2}}$ shell, so that, ($0f_{\frac{7}{2}} - 0f_{\frac{5}{2}}$) shell occurs lower than ($1p_{\frac{3}{2}} - 1p_{\frac{1}{2}}$) shell. Though this assumption can be very crucial for the fine structure of many nuclei, but we expect its effect to be small for our present study since the contribution of spin density term is rather small.

Table 2. Values of constants a, b and c in (21) and (22) for colliding nuclei belonging to differen shells

Colliding nuclei (shells)	V_{JB} c	R_{JB} a_{JB}	b_{JB}	R_{J0} a_{J0}	b_{J0}	R_{JL} a_{JL}	b_{JL}
$0d+0d$	1.5750	4.74	1.07×10^{-3}	3.68	8.87×10^{-4}	9.62	2.26×10^{-3}
$0f+0f$	1.1333	6.19	3.22×10^{-4}	4.77	2.88×10^{-4}	10.63	8.29×10^{-4}
$1p+1p$	1.3920	5.51	1.51×10^{-4}	4.71	1.21×10^{-4}	11.66	2.93×10^{-4}
$0g+0g$	0.8983	7.62	1.04×10^{-4}	5.96	8.29×10^{-5}	14.55	7.62×10^{-5}

Puri and Gupta [4] have characterized the spin density potential by the following four points (shown in Fig. 2 for one reaction):
(i) the height V_{JB} of the repulsive maximum.
(ii) the position R_{JB} of V_{JB}.
(iii) the position R_{J0} where the spin-density potential changes its nature from repulsive to attractive and hence becomes zero; and
(iv) the limiting distance R_{JL} where $V_J(R)$ goes to zero. Following [4], we take $V_{JL} = 0.003$ MeV for practical purposes.
In terms of the above four points, we find that the spin-density part of interaction potential for all nuclei can be expressed analytically by the following simple formula :

$$V_J(R) = \begin{cases} V_{JB}\,\exp\left[\ln\frac{V_{JL}}{V_{JB}}\left(\frac{R-R_{JB}}{R_{JL}-R_{JB}}\right)^{\frac{5}{3}}\right] & for\ R \geq R_{JB} \\ V_{JB} - V_{JB}\left(\frac{R-R_{JB}}{R_{J0}-R_{JB}}\right)^2 & for\ R \leq R_{JB}. \end{cases} \quad (19)$$

with the parameterized expressions for four constants V_{JB}, R_{JB}, R_{J0} and R_{JL} obtained in a similar way as reported in [4]. In ref. [4], the variation of V_{JB} was found to be smooth with a quantity called "Particle-Strength", which is defined as

$$P_s = \sum_\alpha \frac{(2j_\alpha+1)}{4\pi}\left[j_\alpha(j_\alpha+1) - l_\alpha(l_\alpha+1) - \frac{3}{4}\right] \pm \frac{n_v}{4\pi}\left[j(j+1) - l(l+1) - \frac{3}{4}\right]. \quad (20)$$

Similarly, all other quantities, measuring the distances like R_{JB}, R_{J0} and R_{JL}, are found to vary smoothly with $(A_1\ A_2)$ i.e. with the product of the masses of the colliding nuclei.
The maximum of the spin density potential, V_{JB}, for all the shells can be represented by

$$V_{JB} = cP_s, \quad (21)$$

whereas R_{JB}, R_{J0} and R_{JL} vary as straight lines:

$$R_i = a_i + b_i(A_1\ A_2) \qquad (i = JB, J0, JL) \quad (22)$$

The constants c, a_i, b_i for the colliding nuclei belonging to (sd+sd), (0f+0f), (1p+1p) and (0g+0g) are listed in Table 2. A comparison of the exact values of V_{JB}, R_{JB}, R_{J0} and R_{JL} calculated from (8) with the analytical formulae (eqs. (21,22)) are shown in Figs. 3 and 4. We find that the exact values match the parameterized values quite nicely. Furthermore, we also find that the slope of V_{JB}, R_{JB}, R_{J0} and R_{JL} decreases as we go to higher shells. In addition, R_{J0} can also be approximated for all shells by $R_{J0} = (1.12 \pm 0.05)A^{\frac{1}{3}}$.

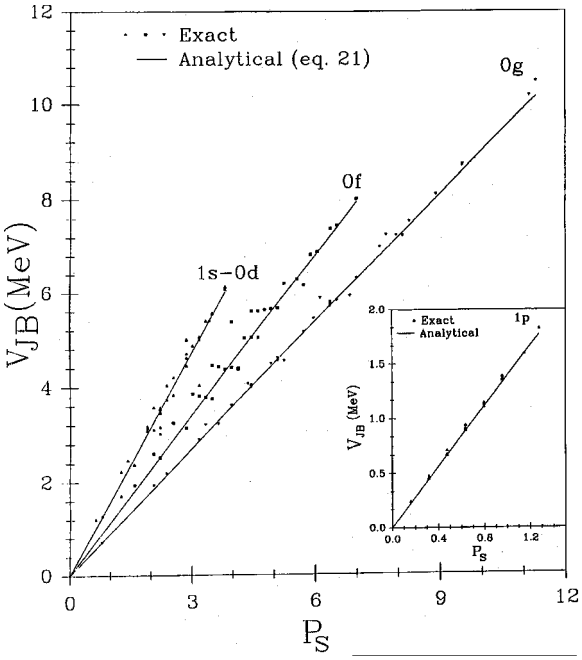

Fig. 3. V_{JB} as a function of "Particle Strength" P_s

This means that the spin-density potential changes its sign from repulsion to attraction at a distance close to the radius of the compound system ($\approx 1.12 \times A^{\frac{1}{3}}$). A comparison of exact ((8), symbols) and the parameterized spin-density potential ((19), solid line) is also shown in Fig. 2 . We find that our analytical expressions reproduce the exact spin density potential very accurately. In most of the collisions reported here, the analytical and the exact results are quite close. Hence, we can use (19-22) to calculate the spin density part of the interaction potential analytically. An application of (19-22) equations is straight forward. Once we know the masses of colliding nuclei, we can find V_{JB}, R_{JB}, R_{J0} and R_{JL} from (20-22) and Table 2. These four parameters finally appear in (19) to generate the spin density potential.
Then using (18) and (19), we can generate the ion-ion interaction potential analytically. This simple parameteri-

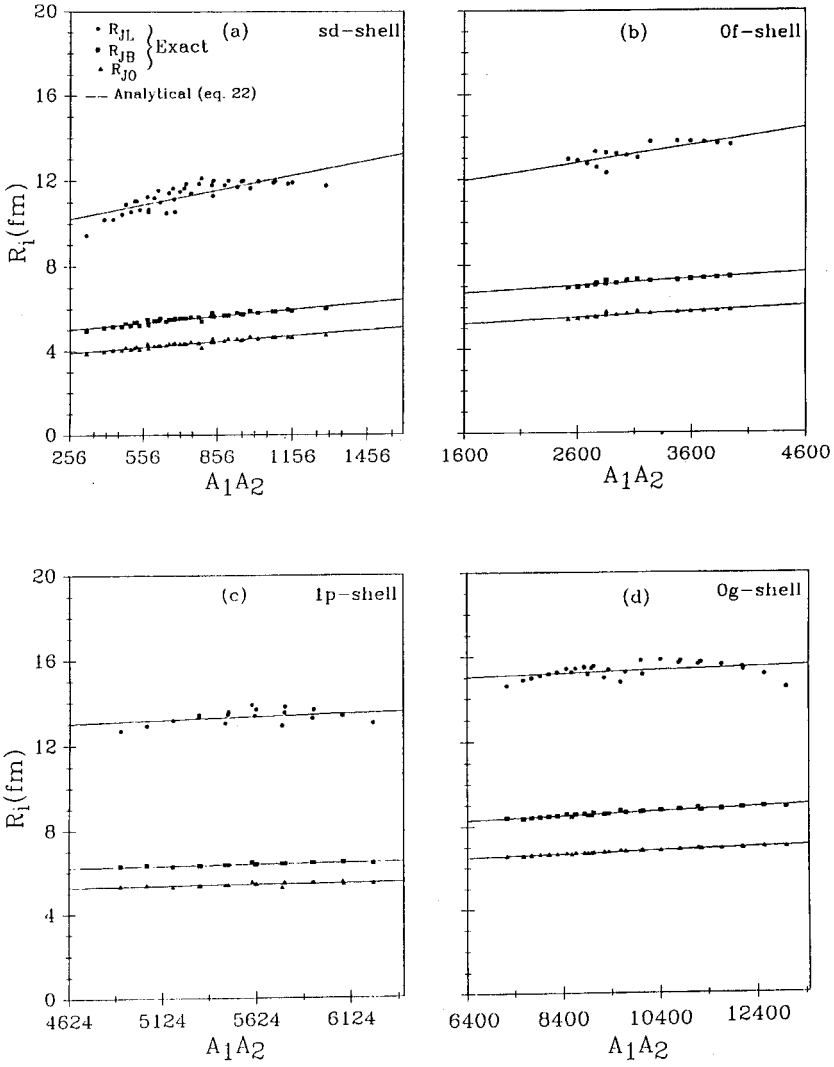

Fig. 4. The R_{J0}, R_{JB}, and R_{JL} as a function of product of masses $(A_1 \cdot A_2)$

zation introduces great simplification to calculate the ion-ion potential using Skyrme Energy Density Formalism. In any heavy ion collision, the masses (and the charges) of the colliding nuclei and the bombarding energy are the inputs of an experiment. One can add the Coulomb interaction (which depends upon the charges of colliding nuclei) to compute the fusion barriers and cross-sections. Our present attempt is a first complete parameterization which includes the spin density distribution of colliding nuclei and is based on a microscopic theory.

The test of the accuracy of our analytical potential is made in Fig. 5. Here we compare our total potential with other theoretical potentials taken from ref.[16]. We find that our potential is very close to that of Krappe et at. and proximity potential. The displayed part is the surface part the ion-ion potential.

To compare and calculate the fusion cross-section, we use sharp cut of model where the fusion cross-section is defined as:

$$\sigma(E_{c.m.}) = \pi R_B^2 \left[1 - V_B/E_{c.m.}\right]. \qquad (23)$$

where R_B and V_B are the barrier position and height [5]. Using our analytical ion ion potential in above formula, we compute the fusion cross-section. In Fig. 6, we show our calculated fusion cross-section for the collision of $^{32}S + ^{27}Al$. The experimental data and other theoretical results are taken from [16]. As SEDF is valid for even even colliding nuclei, we show the results for $^{32}S + ^{28}Si$ and $^{32}S + ^{26}Mg$. We find that our results are in very good agreement with experimental data and also with theoretical results of Krappe et. al, and proximity potental. It is worth to mention that we have calculated the fusion cross-section for several different reactions and our results are in good agreement with existing experimental data. This shows the validity of our analytical ion-ion potential.

IV Summary

We have presented a calculation of the heavy ion potential using Skyrme energy density formalism. To parametrize the ion-ion potential, several hundred collisions were generated involving (1s-0d) to 0g-shells. The spin-independent

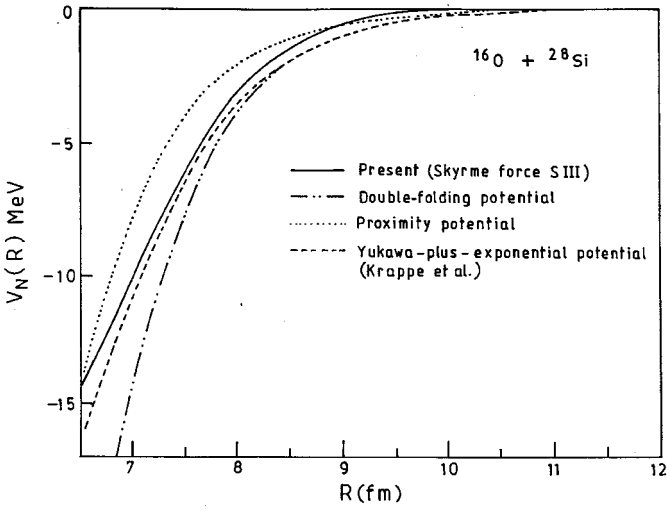

Fig. 5. The parametrized ion-ion potential along with other theoretical potentials as a function of the saparation distance R. The theoretical results of proximity potential, Double - folding potential and Krappe-Nix-Sierk potential are taken from [16]

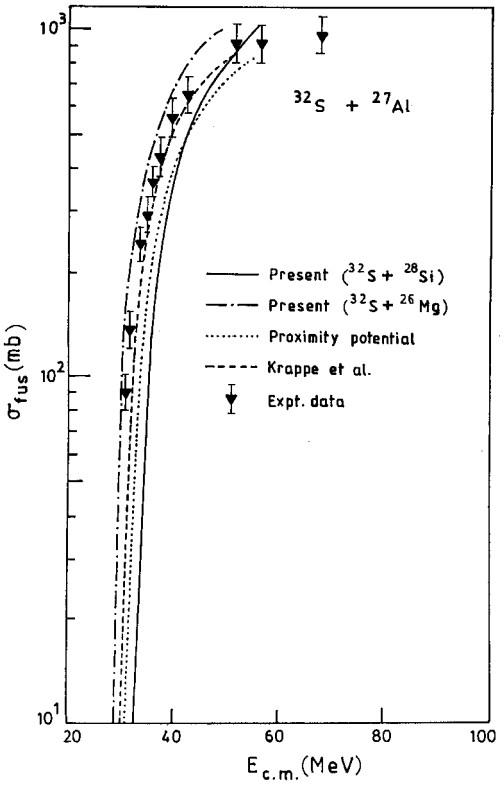

Fig. 6. The Fusion cross-section as a function of the centre of mass energy $E_{c.m.}$. Here our present results are shown for the collisions of $^{32}S + ^{28}Si$ and $^{32}S + ^{26}Mg$. The fusion cross-sections using proximity potential and Krapp-Nix-Sierk potential are extracted from [16]

part of the ion-ion interaction potential is parameterized in the spirit of proximity theorem and a simple analytical parameterization of spin-dependent part of the ion-ion potential is presented in terms of the masses of colliding nuclei and a quantity called particle strength. This analytical formulation introduces a great simplification for heavy ion potentail calculations based on a microscopic picture. The parametrized ion-ion potentail is in good agreement with other theoretical potentials. The fusion cross-section calculated with above analytical formulation is very close to the available experimental data.

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